Identifying Minor Planets in UCAC Data and Deriving Accurate O – C

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ABSTRACT

With growing capability to observe fainter celestial bodies, the observation of minor planets, or asteroids, is a recent addition to star catalogs. The US Naval Observatory (USNO) Charge-Coupled Device (CCD) Astrograph Catalog (UCAC) has observational data (O) on minor planets. Customized software was used to calculate ephemerides (C), using the orbital elements from Jet Propulsion Laboratory (JPL). The ephemerides provided the location of minor planets for the approximate time of UCAC observation. The present study involved development of software to identify minor planets in the ephemerides and match them with the unidentified celestial bodies in UCAC observations, using the minor planets' coordinates. Mean motions of minor planets from the ephemerides were used to make adjustments to minor planets' coordinates based on the time difference between the UCAC observation and ephemeris calculation. The output tables and plots were derived using object-oriented and procedural programming designs based on the strengths of each design. A statistical analysis of the spread of minor planets and the observed minus calculated, $O - C$, was conducted for minor planet positions and magnitudes. With precise error determinations, possible orbital improvements can be made to determine asteroid collisions with other matter, including Earth, well in advance.

Subject headings: minor planets, astrometry, UCAC

1. INTRODUCTION

There is an abundance of precise observations in the United States Naval Observatory (USNO[1]) Charge-Coupled Device (CCD) Astrograph Catalog (UCAC) of faint celestial bodies, as the star catalog has observations of celestial bodies within 8th and 16th magnitude[2] (1). Minor planets, or the asteroids, in the solar system of the sun are included in this data. It is thought practical to identify these unidentified minor planets in UCAC observations because such detailed information on each minor planet was previously not available. The identification of minor planets is done by matching the coordinates of unidentified celestial bodies in the UCAC observations for each of its CCD frames with the coordinates of identified minor planets in ephemerides which provide the calculated position of minor planets, provided that the difference between the coordinates of these two sources is within a tolerance of 5 milliarcseconds (mas). The ephemerides have the minor planet names used to identify unidentified minor planets in the UCAC data.

NASA's Jet Propulsion Laboratory (JPL) provided the orbital elements for the minor planets, which are physical constants describing the motion and path of minor planets in the solar system (2). These orbital elements were used to compute the ephemerides, known as C because they are calculated. The ephemerides provided the minor planets that were in the sky during the time of UCAC observation. The UCAC observations are known as O because they are observed.

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[1]For a full list of acronyms and other symbols, see Table 5.

[2]Magnitudes discussed here are all apparent magnitudes. Greater magnitude of a celestial body indicates greater faintness of that body when observed.

Systematic errors are biases in the determination of observed and calculated data, while accidental errors are unique errors only pertaining to the current data (2). As UCAC observations are precise to 15-70 mas and have less than 10 mas of systematic errors, minor planets matched to the ephemeris data are expected to have slightly different coordinates and magnitude (1). To measure this difference, $O-C$, the observed minus calculated, provides a model which evinces accidental and systematic errors and (3). This model is a sampling distribution, as not all minor planets are observed, but only a sample. While $O-C$ errors come from errors in observation, errors in constants used in calculation, and errors in theory used for computation, this study focuses only on observational and computational errors rather than on theory (2). The appropriateness of using UCAC data in future orbital improvement or other calculation is determined based on analyzing the errors involved in the data.

One purpose of this research to identify minor planets in UCAC data by matching identified minor planets in the ephemerides which are based on JPL orbital elements with celestial bodies in UCAC observations. The other purpose is to determine accidental and systematic errors in data to speculate on the feasibility of using UCAC minor planet data for other purposes. Accidental errors are judged to improve future matchings of minor planets between observed and calculated data regarding the difference tolerable between the two datasets for a match, while systematic errors are identified to improve both the original observed and calculated data sources. For follow-up studies, these identifications would be useful for minor planets' orbital improvement.

1.1. UCAC Project

The USNO conducted a survey of all sky to compile the comprehensive star catalog, the UCAC. It made use of a 4K charge-coupled device (CCD) to digitally photograph the celestial bodies in the sky, and store each photograph as a CCD frame (1). The fields[3] are one square degree range approximately from -90 to 40 declination (1). Minor planets, usually the brighter ones from 12th to 14th magnitude, are included in the UCAC frames because their magnitudes fit within the 8th to 16th magnitude range of UCAC observations (1).

The UCAC project was conducted between 1998 and 2002 for the second release of data. Data was collected at Cerro Tololo Inter-American Observatory (CTIO) and Naval Observatory Flagstaff Station (NOFS) afterwards. The observational data in the UCAC includes the position and magnitude of celestial bodies and descriptive information on the conditions for each CCD frame, including the exposure times from beginning to end of observation of each frame.

The identification of minor planets in UCAC data allows for them to be taken out of the catalog of only star reductions and gives them a separate category of observed data to experiment on. Currently, reductions are being done on sets of stars, so identifying minor planets helps reduce errors in those as well.

[3]Field data is taken from the second UCAC release (UCAC2)

1.2. Further Applications to Astrometry

Astrometry is a subject where precision improvement in positional data comes with better understanding of technology used for observation. Understanding data among the fainter stars is a slower process, as bright stars are used as reference stars. For UCAC data the Tycho-2 catalog is used, which is only on the 10 - 100 mas precision level (per star coordinate). The Tycho-2 catalog is on the same coordinate system as the Hipparcos catalog, which was on the 1 mas level in 1991, but has errors of 10-15 mas now (4). UCAC's coverage of the majority of fainter celestial bodies have a precision (1 σ) of 20 mas for the 10th to 14th magnitude range, so systematic errors must be kept at a minimum (1).

Part of the systematic errors include the availability of proper motions, which provide the movement of celestial bodies in the celestial sphere over time. In the case of minor planets, the movement is recognized as mean motion, because the term "proper motion" is generally reserved for stars. This is because there are too many minor planets for their radial velocity to be determined, meaning that a radial velocity and proper motion cannot be combined to form the space-motion vector best describing the motion of the celestial bodies (2). Hence, since mean motion is all that was available to correct for differences between data, there could be more investigation to find more positions of minor planets to account for and reduce positional errors in orbital correction.

When accidental errors are identified, false identifications of minor planets decrease because there is less error in matching. When the reduction systematic and accidental errors are combined, minor planet orbits can be significantly improved, thus improving the capability to detect collisions with other matter including Earth.

2. PROGRAMMING PROCEDURES

2.1. Data Used

In order to find differences between calculated ephemeris data and observed celestial body data, common ground regarding time scale and celestial body location had to be found. Though conversions would be made in the programs written, the variables similar between the two datasets were time of observation, minor planet right ascension, α, and minor planet declination, δ, where α and δ constitute the absolute location of the minor planets on the celestial sphere. The conversions and corrections to data are discussed in the procedures.

2.1.1. Ephemeris Data

Minor planet ephemerides were made available due to orbital elements found by researchers at NASA/JPL, who collected observations of minor planets and derived orbital parameters. These orbital elements from NASA/JPL were used with a customized version of Project Pluto's Guide 6.0 software, which had a special plugin for a separate executable developed for it in order for Fortran-style tables of text with columns to be generated. These tables provided the position in the sky at the specific times in the night during UCAC observations. Minor planet positions were calculated for the middle of the night for when data was collected at both of the observatories of CTIO and NOFS for the UCAC. Hence, the ephemeris data is known in this research as the calculated data.

Minor planet ephemerides were limited in magnitude range to better reflect what could have been observed in the UCAC, as is seen in Figure 1 and Table 1.

Table 1: Limits to Minor Planets Calculated in Ephemerides

Parameter	Minimum	Maximum
Magnitude	5.9	16.8
Right ascension [h]	0.000	24.000
Declination [°]	-58.7227	82.852
Universal time [h]	0.000	24.000
Modified Julian date [day]	1000	3145

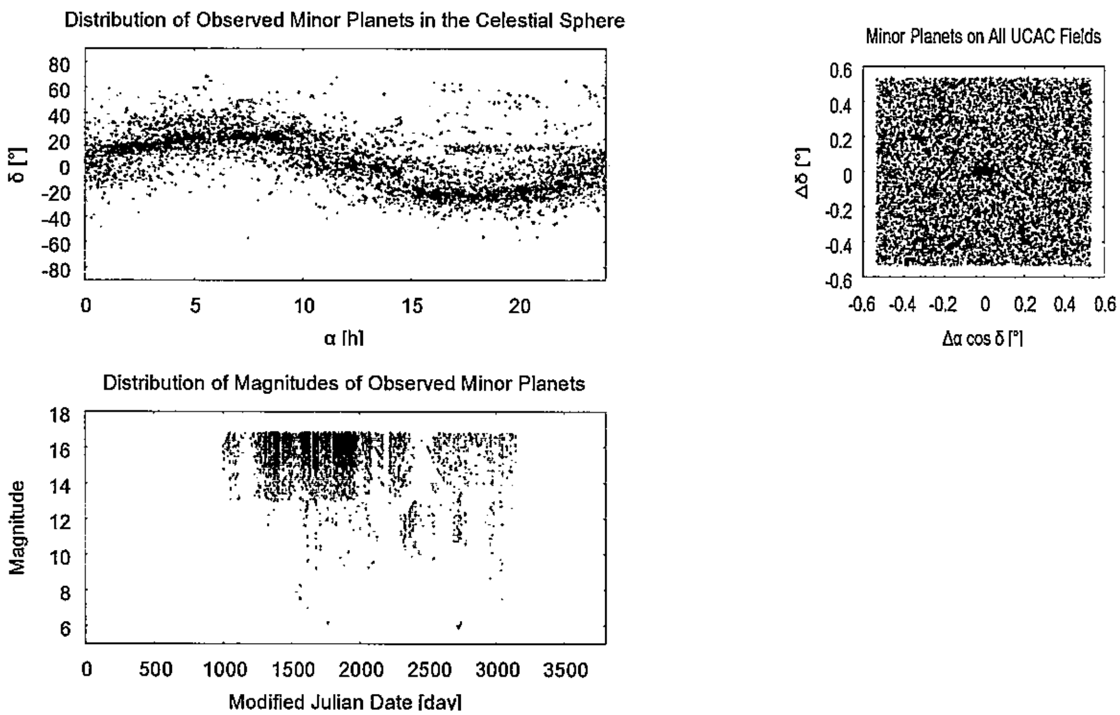

Fig. 1.— Distribution of minor planets in the ephemerides. [These charts show the limitations placed on the calculation of ephemerides, such as the limiting of minor planet magnitude to 16.5 mag.]

2.1.2. UCAC Data

The observed star catalog data from the UCAC comes directly from the final position files providing the celestial sphere position, observation exposure time, and exact observing time. This data was made available partly from a file containing celestial sphere positions for all detected objects in a given CCD frame. Then, another file provided metadata on the astrophotography of each frame including exposure time (ET) and universal time (UT), and is referred to by the previous file.

2.2. Programs for Comparison and Identification

The programming design was conducted thoroughly such that the code could be reused on the final data release of the UCAC and future star catalogs. Capabilities in the programming languages of C++ and Fortran were analyzed for their features, as well as the hardware capabilities of computers used for astrometry. It was adopted that the programs written to identify minor planets would be developed on Linux using standardized languages usable on various platforms. All input data was given in text format, which is read into variables in column-major order by Fortran and C++ programs. These capabilities resulted in the choice of both of those languages in designing the programs.

For most input/output programs with little math involved, Fortran was found effective because of its optimization to handle formatted and unformatted data in few statements (5).

2.2.1. Coding

Figure 2 shows the flow of data in each program. A Fortran program checks to determine the number of UCAC fields and whether they are designated as a test for minor planets. The main C++ program, written with object-oriented programming (OOP), scans in UCAC field data and Guide ephemeris data. This data is stored in dynamically linked lists for better memory management, to be accessed via accessor methods in classes representing field data and minor planet data (6).

In the C++ code, a `FieldSet` class has public methods allowing access to a private `list` of `FieldData` objects. The `FieldData` objects give public access to the name of frame and field, α_{min} and α_{max} based on declination, central coordinates α and δ; $\Delta\alpha$ cos $\delta = \Delta\delta$ $= 0.5°$, and modified Julian date (night number, JD) and universal time (time of night, UT) during capture with exposure time. Privately fields check whether α is on a split domain ($\alpha_1 >> 23.5$ h, $\alpha_2 << 0.5$ h) or whether declination has a range over a celestial pole ($\delta_1 >>$ $89.4°$, $\delta_2 >> 89.4°$).

A `MPSet` class has public methods to access and traverse a list of minor planets represented as `MPData` to access its name, magnitude, JD, UT, and α and δ adjusted for mean motion differences using $\Delta\alpha = \mu_\alpha \Delta t$ and $\Delta\delta = \mu_\delta \Delta t$. A method also checks whether it belongs in a given field.

The C++ main procedure generates these sets to analyze and output a raw table giving sorted information on the UCAC fields and the associated minor planets. Shellscripts and Fortran programs process this table for more output tables and plots with Gnuplot giving what is described in Figure 1 and Table 1.

The final position identifier program, written in Fortran, converts all units to the UCAC format: milliarcseconds for coordinates and $\frac{1}{1000}$ of the magnitude scale. Then it checks each matched minor planet to identify candidates within the frame.

2.2.2. Corrections to Ephemerides and Sorting by Field

In the first program represented by the left flowchart in Figure 2, minor planets in the ephemerides are mapped to the UCAC fields they would be found in. Each field has a center in the celestial sphere, covering $\Delta\alpha\cos\delta = \Delta\delta =$ [-32 arcmin, 32 arcmin] from there. Notable here is that corrections to the ephemerides' minor planet positions are made. The calculated position corrected for time differences in ephemeris data later to be compared with raw UCAC data, when observation exposure time (ET) and difference in Universal Time ($\Delta UT = UT_{UCACframe}$ - $UT_{Ephemeris}$) are known, is given by:

$$\alpha = \alpha_{Ephemeris} + \mu_\alpha(\Delta UT + \frac{1}{2}ET) \quad (1)$$

$$\delta = \delta_{Ephemeris} + \mu_\delta(\Delta UT + \frac{1}{2}ET) \quad (2)$$

The first C++ program thus ensures the correct calculation of each minor planet's position by adding or subtracting the time difference between calculation and observation and adding half of the exposure time to reflect the exact position which the UCAC observation would have if it were that minor planet.

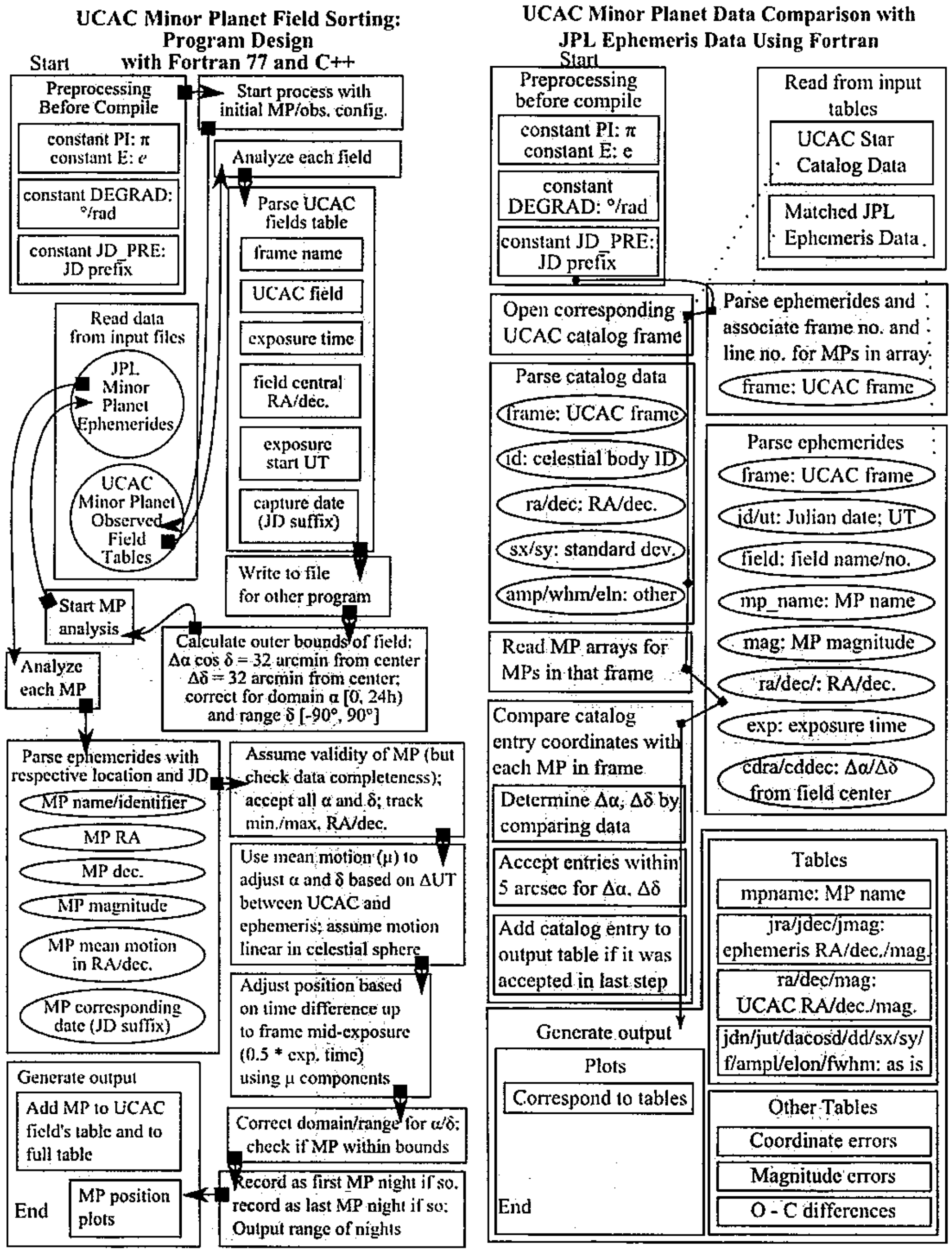

Fig. 2.— Components of the minor planet matching and sorting algorithm on a flowchart. [The left side shows the first program matching minor planets in ephemerides with UCAC frames while correcting positions. The right side shows that matching of the matches of the left side program with the final position of celestial bodies in UCAC observations. Abbreviations and other shortenings are listed in Table 5.]

2.2.3. Comparison and Identification

When the table matching ephemerides to frames is made, the second program represented with the right flowchart in Figure 2 compares each minor planet to all celestial bodies in the given frame to identify if they are minor planets. Taking the matched UCAC frames with the minor planets in the ephemerides, the unformatted UCAC data was matched with the minor planets corresponding to the UCAC frame.

2.2.4. Testing

To determine whether an object with a calculated position belonged in a field it had to fit in $\Delta\alpha \cos\delta = \Delta\delta = 0.5°$from the center of the field. The intervals [0, 24 h) applied for α and [-90°, 90°] applied for δ (7). For declinations out of this range, the amount it was out of range by was subtracted from the extremum. For right ascensions out of this domain, 24 h was added or subtracted to get the coordinate in domain. Most debugging and testing time was spent to ensure that the program met these special cases. The output table was checked for accuracy manually with test cases ranging from basic minor planets in the table in different α and δ zones. as well as minor planets found to be in certain fields. These were manually checked for presence in the output table. The intervals of coordinates α and δ were checked using `assert` statements in the C++ program at various points. Borderline data near the endpoints of these intervals was also checked manually.

Iteration through lists of fields and minor planets was found to be quite effective, though tedious to initially implement due to the need to correctly transfer references from classes to the `main` method. Shellscripts were later found to be useful for customized output using GNU `awk` and `sed`, but not for checking conditions in data.

Both Fortran and C++ programs handled formatted I/O as expected provided that the formatting statements were meticulously created.

2.3. Methods to Analyze Data

Techniques for determining errors using $O-C$ charts are used for analysis. $O-C$ sampling distributions are found for minor planets' right ascension (which is multiplied by the cosine of the declination to counter the effect of being less spaced out at the celestial poles), declination, and magnitude.

Standard error, s, is used to determine the variation in the sampling distributions of comparisons between UCAC data and ephemerides, thus indicating relative variances in the data in general (8). The intermediary used to determine standard error is standard deviation, σ_s, which is a more general root-mean-square measure of variation in the sample or population (2). Standard error is given by:

$$s = \frac{\sigma_s}{\sqrt{n}} \tag{3}$$

$$\sigma_s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \tag{4}$$

where σ_s is the sample standard deviation, n is the size of the $O-C$ dataset, x_i represents an individual value, and $\bar{x}$ is the arithmetic mean of the $O-C$ dataset (8).

To determine the probability of the occurrence of error, P, another formula specific to the $O - C$ diagram is used. This probability is based off of the index of precision, h, which is inversely proportional to the standard deviation. This is useful eventually towards the application of the least squares method in determining weights of values in finding a best fit model of the data (2). The probability of occurrence of sets of errors is given by:

$$P = \frac{h^n}{\pi^{\frac{n}{2}}} e^{(-h^2 \sum_{i=0}^{n} x_i^2)} \prod_{i=1}^{n} \Delta x_i \tag{5}$$

$$h = \sqrt{\frac{1}{2\sigma_s^2}} \tag{6}$$

where x_i is an individual error represented by an $O - C$ value for the observation O_i (2). As is seen in this equation, errors are summed up in the exponent and the differences between errors are multiplied by this value in the end. Because the function of the probability of occurrence negates the exponent, which has the sum of the errors, the places where this function has its greatest values are where the least errors are likely to occur.

The probability of error occurrence function is used to determine an appropriate tolerance for minor planet matches between the UCAC observations and ephemeris calculations.

3. PRESENTATION OF RESULTS

OOP proved effective in the initial frame matching, due to the great quantity of data and need to make several type conversions. For the matching of the final positions with minor planets with calculated positions belonging to the frame, a simple Fortran program did the job since units of the positions were the same.

The input data had some variability. For each night, between 500 to 2000 minor planets visible within range were provided in the ephemerides. UCAC observations had between 0 and 2000 candidates for minor planets, approximately, for each night.

3.1. Identified Minor Planets

A total of 26,947 minor planets were matched with a UCAC CCD frame using ephemeris data in the first step. The distribution of these calculated minor planets was shown in Figure 1. 4,041 CCD frames of the 273,746 total in all UCAC data actually contained an identified minor planet. 1,622 distinct minor planets were matched with one or more celestial bodies in UCAC data, using the UCAC CCD frames they belong in. This was considered the identification of minor planets in UCAC data. There were 22,199 different minor planets in the calculated ephemerides.

Counts for the number of identifications found for each minor planet are in Figure 3.

3.2. Output Table Format

The final identification output table is shown in Table 2. It provides each celestial body matched as a minor planet. All of the data except the name and one magnitude value comes from UCAC observations. Two other columns indicate the $O - C$ as $\Delta\alpha \cos\delta$ and $\Delta\delta$.

Table 2: Sample Minor Planet Identification in UCAC Data

mpname	frn	jd	mput	ra	dec	mag	mpmag	. . .
1999_JF52	148234	1846	2.72833	20504886	375033960	15910	16500	

. . .	dra	ddec	sx	sy	ampl	fwhm	elon	dacosd	dd
	-55	876	44	46	220	1736	1414	0.083	0.168

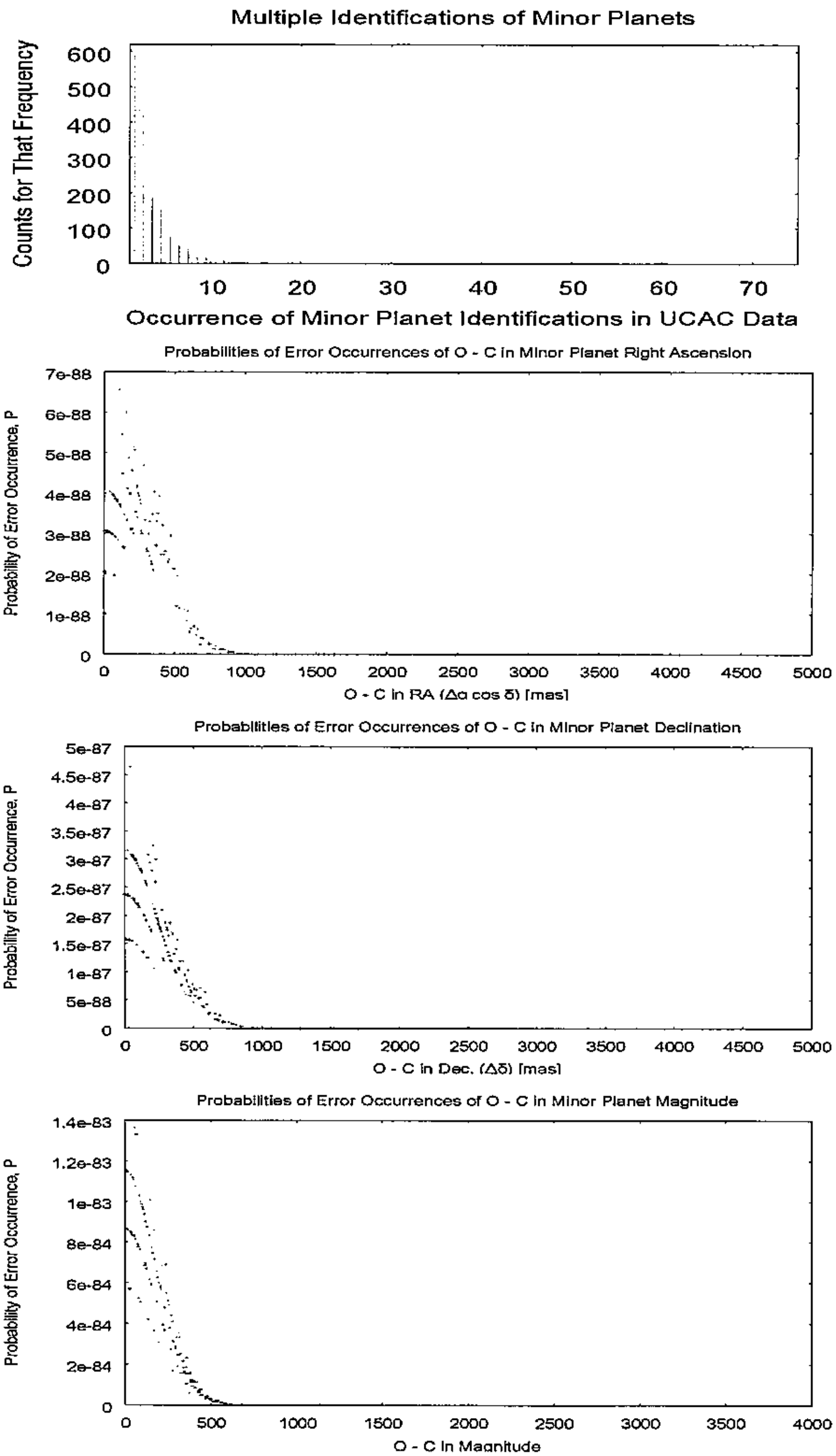

Fig. 3.— Minor planet identification errors. [1st chart gives times a minor planet was identified. Other charts give the probability of error occurrence for the $O-C$ errors for each type of $O-C$ diagram.]

Table 3: Quantities and Units for Output Table

Description	Column name	Units	Quantity
Minor planet name	mpname	none	
UCAC Frame number	frn	none	
UCAC JD Night number	jd	night number	
Frame Observing Time	mput	[h]	
Right ascension, α	ra	[mas]	
Declination, δ	dec	[mas]	
UCAC Object Magnitude	mag	magnitude scale	
Ephemeris Magnitude	mpmag	magnitude scale	
Eph. - UCAC	dra	[mas]	$\Delta\alpha \cos \delta$
Eph. - UCAC	ddec	[mas]	$\Delta\delta$
Standard error, x, y	sx, sy	[mas]	
Stellar profile amp.	ampl, fwhm	pixelcount	
Image elongation	elon	1/1000 elongation	
UCAC FieldCenter - Eph.	dacosd, dd	[deg]	$\Delta\alpha \cos \delta$, $\Delta\delta$

4. ERROR ANALYSIS USING $O - C$

The $O - C$ differences are given in Figure 3, whose descriptive statistics are given in Table 4. The methods used to get these values are discussed in the procedures.

Table 4: Descriptive Statistics for Each $O - C$ Dataset

$O - C$ Dataset	Mean, $\bar{x}$	Sampling Standard Deviation, σ_s	Standard Error, s
Magnitude	-263.495	876.700	13.223
α	71.698	1393.974	21.025
δ	112.143	1275.756	19.241

Systematic errors are drawn from observations of the $O - C$ diagrams (Figure 4). From the slightly cyclic but mostly spread out behavior of the $O - C$ diagrams, it can be gathered that an interdependence bias is in the data in which further point-spread-based modeling must be done to find errors (9).

Accidental errors seen in the probabilities of error occurrence (Figure 3) show that for both $O - C$ in right ascension and declination should be limited to 800 mas in the matching, because most errors are between 0 and 800 mas for those charts. A magnitude constraint

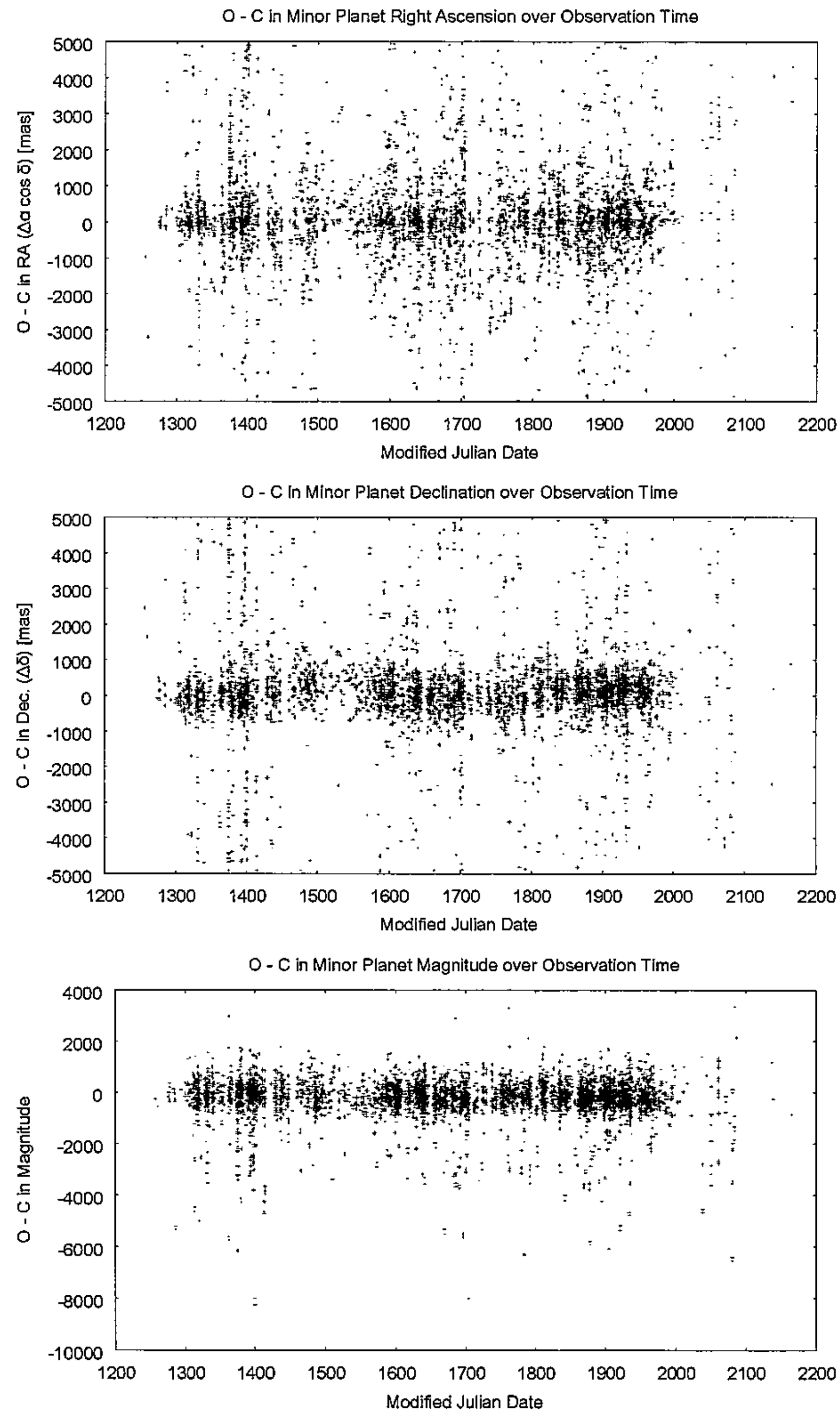

Fig. 4.— $O - C$ Diagrams in Minor Planet Position (Right Ascension and Declination) and Magnitude

could also be added as most errors on that chart are within 0.7 mag.

5. CONCLUSIONS

Minor planets were successfully identified, and the $O - C$ diagrams did show close scattering such that reasonable limits to error could be determined for future matching. This is true especially for the $O - C$ in magnitude. Since there are several outliers with values entirely out of the scale of the remainder of magnitude values, a logical explanation of this is that minor planet identifications were not strictly limited to a small area. Hence, some bright stars could have been inappropriately identified as minor planets. 1 arcsecond is a reasonable limit to the tolerated $O - C$ error in the matching process. Applying these changes to the matching process would significantly reduce accidental error, making minor planet identifications more precise.

Systematic errors were also found, but general enough to allow for future investigation. The interdependence biases found require the use of deconvolution methods involving making complex fits and having more knowledge on the conditions during observation and the methods to get the constants–the orbital elements–of position calculation than was available for this research (9).

Modular programming design was effective, though debugging time increased with task and language variety. Fortran and C++ were useful for formatted input and output. C++ was also useful for calculations on components as dynamic objects.

It took the most time to correctly handle data between the two given sources. Although ephemerides had minor planet positions calculated to a time fairly close to the time of observation, mean motions according to the ephemerides were used to correct for differences in time between the ephemeris position calculation and UCAC observation. After correcting for this time difference, another smaller time difference was accounted for. This was the mid-exposure time, which represents the time when the coordinates observed truly represent what they are for that time in the observation.

The $O - C$ data here can still be used for orbital improvement of minor planets, entailing an improvement of the accuracy of the mass of minor planets, which in turn will help toward being more accurate about minor planet positions and movement to avoid collisions with spacecraft or even planets such as Earth. The identification of accidental errors here helps toward achieving this goal, but more systematic error analyses need to be done for the most improvement.

One critical component to the programming design causing some error was the extrapolation of observation data to ensure that minor planets in the UCAC data or ephemerides are compared at the same timeframe. Since mean motions in minor planets are applied to the observed data to bring compared coordinates to the same timeframe, there could be errors in the analysis of errors because these motions are not constant with different coordinates on the celestial sphere.

6. ACRONYMS

Table 5: Aliases, Acronyms, and Abbreviations

DEGRAD	1 deg = 1°= π / 180 rad = 60 arcmin = 3600 arcsec		
CTIO	Cerro Tololo Inter-American Observatory (Chile)		
NOFS	Naval Observatory Flagstaff Station (Arizona)		
CCD	charge-coupled device	obs.	MP field observations
no.	number	PI	$\pi = 3.1415926535897932$
dec.	declination, δ	RA	right ascension, α
E	$e = 2.7182818284590452$	STDERR	standard console error output
JD	Julian date	STDOUT	standard console output
JD_PREF	JD prefix	UCAC	USNO CCD Astrograph Catalog
JPL	Jet Propulsion Laboratory	USNO	United States Naval Observatory
MP	minor planet	UT	universal time

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